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Regions of Acceptability Using in Reliability-Oriented Design

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***Abstract*— A problem of acceptable regions construction and their utilization at design of analog engineering systems is considered in this paper. The methods of multidimensional probing on a regular grid and multidimensional figure approximation with a set of parallelepipeds are used for region of acceptability construction. A parallel algorithm of region of acceptability construction is described. Software tools for construction and utilization of regions of acceptability in reliability-oriented design are described.**

***Keywords*— parameter; region of acceptability; reliability; parallel algorithm; engineering system design.**

I. Introduction

The problem of feasible parameter region exploration often arises at engineering systems design. This problem is associated with a set of specific procedures such as parameters tolerancing and choosing their nominal values, estimation of system sensitivity and reliability. As a rule, a feasible parameter region (region of acceptability or RA) is a domain comprised of parameter vectors which yield proper system performance. Obtaining the region characteristics or its approximation significantly facilitates solutions of design tasks associated with reliability control. Essential difficulty of the region approximation consists in high dimension of parameter space, incomplete prior information and only pointwise exploration of parameter space with system performances calculation.

There are different methods for constructing RA. The method of approximation with a hyper-parallelepiped is in general use [1.2]. The methods of approximation with ellipsoids and polytopes are more advanced, but more complicated [3]. One more approach consists in approximation with discrete set of elementary figures. Usually, hyper-parallelepipeds are used as elementary figures [4]. In our paper, this method of approximation is applied to determine RA.

The process of a multidimensional region construction with a discrete set of elementary parallelepipeds (boxes) is associated with problems of large amount of data to be processed and stored. The task of RA construction is almost incogitable without high performance parallel computations. The solution of these problems generally consists in data splitting and application of data compression algorithms.

The application of parallel computing requires decomposition of a task. In this work, it is shown that the decomposition of the process of acceptable region construction can be carried out by splitting data to be processed in parallel. The model of the region approximation allows splitting the data into various parts of various volumes dynamically.

This work is devoted to consideration of RA construction problem and application of RA to reliability-oriented deign. It is shown how RA can be used for system parameter sizing at design process. Software tools for solving these tasks are introduced.

II. Region of Acceptability

A. Region of Acceptability Definition

From a consumer point of view, a system has its performance characteristics (average power, delay, gain, etc.). The performances are given as m-vector (1):

$$\mathbf{y} = (y_1, y_2, ..., y_m)\,. \tag{1}$$

From a design point of view, any system consists of elements/components that perform their functions. These elements are considered to be atomic. Thus, system parameters are considered as the n-vector:

$$\mathbf{x} = (x_1, x_2, ..., x_n)\,. \tag{2}$$

System performances (1) depend on parameters (2) of system elements (system parameters). System topology is defined by the model (3) which relates system parameters (2) to the system performances (1):

$$\mathbf{y} = \mathbf{y}(\mathbf{x})\,. \tag{3}$$

System components are influenced by different factors like ambient temperature, supply voltage, radiation, etc. These factors are usually taken into account in the model (3) as operational parameters and cannot be controlled by the designer. Operational parameters and aging factor cause deviations of system parameters which, consequently, cause system performances deviations. Usually, system performances (1) are constrained by performance specifications (4):

$$\mathbf{y}_{\min} \le \mathbf{y}(\mathbf{x}) \le \mathbf{y}_{\max} . \quad (4)$$

The deviations of system parameters may cause violation of performance specifications (4) that means system failure. The task of parametric synthesis [1] as one of design stages consists in nominal parameters choosing to meet the performance specifications (4) with the account of system parameter deviations during operating cycle. The solution of this task is often associated with determination of a region of acceptability as defined in (5):

$$D_x = \{\mathbf{x} \in \mathbf{R}^n \mid \mathbf{y}_{\min} \le \mathbf{y}(\mathbf{x}) \le \mathbf{y}_{\max}\} . \quad (5)$$

The region of acceptability and schematic illustration of system parameter deviation from nominal values $\boldsymbol{x}^0$ at the moment t_0 to gradual parametric failure at the moment t_2 are presented in Fig. 1.

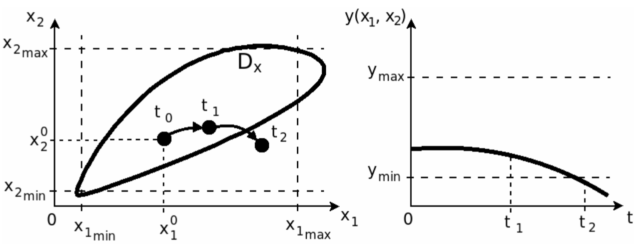

Figure 1. The region of acceptability and gradual parametric failure

B. Grid Approximation of a Region of Acceptability

As it was said before, the approximation of an n-dimensional region with a discrete set of elementary hyper-parallelepipeds (boxes) is used in this work. The basis of such representation of a region is an n-dimensional regular grid inside a bounding box (circumscribed box [1, 4] or parameter tolerances box) defined by the constraints (6):

$$\mathbf{x}_{\min} \le \mathbf{x} \le \mathbf{x}_{\max} . \quad (6)$$

The grid nodes (7) define corners of the elementary boxes:

$$x_{ij} = x_{i0} + j \cdot h_i , \quad (7)$$

where $i = 1,2,...,n$ is an index of a parameter, $j = 0,1,...,q_i$ is the node index for i-th parameter (the first node $x_{i0} = x_{i_{\min}}$), $h_i = (x_{i_{\max}} - x_{i_{\min}}) / q_i$ is the grid spacing for i-th parameter, q_i – is the amount of "quanta" – the atomic intervals into which the range $[x_{i_{\min}}, x_{i_{\max}}]$ is divided. For the each x_i inside a "quantum" $\partial \mathbf{y} / \partial x_i = 0$ is supposed. Every "quantum" is indexed with $k_i = 1,2,...,q_i$, thus the set of indices $(k_1, k_2, ..., k_n)$ identifies an elementary box. It is supposed that every point $\mathbf{x}$ inside an elementary box yields the same performances as its central point $\mathbf{x}_c(k_1, k_2, ..., k_n)$ with the coordinates defined in (8):

$$x_{i_c}^{k_i} = x_{i0} + k_i \cdot \frac{h_i}{2}, \ \forall i = 1,2,...,n . \quad (8)$$

Every point $\mathbf{x}_c(k_1, k_2, ..., k_n)$ acts as a sampling point for elementary box identified by the indices $(k_1, k_2, ..., k_n)$. System performances (1) are calculated for every elementary box's sampling point using the model (3). Then these performances are compared with performance specifications (4). Thus, for every elementary box, the binary function (9) is calculated:

$$F_{D_x}(k_1, k_2, ..., k_n) = \begin{cases} 1, \ \mathbf{y}_{\min} \le \mathbf{y}(\mathbf{x}_c(k_1, ..., k_n)) \le \mathbf{y}_{\max} \\ 0, \ \mathit{otherwise} \end{cases} . \quad (9)$$

The function (9) determines the membership of a sampling point in the region D_x . Let us denote the set of elementary boxes B_g , then the function (9) defines a partitioning (10) of this set:

$$B_g = B_g^0 \cup B_g^1 , \ B_g^0 \cap B_g^1 = \varnothing . \quad (10)$$

The subset B_g^1 is the approximation of the region of acceptability D_x , constructed with a discrete set of elementary boxes, defined with a regular grid.

The region of acceptability approximation with a grid is defined with the model (11):

$$G_R = (n, B, Q, S), \quad (11)$$

where n is the amount of designable system parameters (2), $B = ((x_{i_{\min}}, x_{i_{\max}}), \forall i = 1,2,...,n)$ is a bounding box, defined by the constraints (6) in system parameter space, $S = (s_1, s_2, ..., s_R)$ is a set of membership indicators that store results of membership function (9). Every indicator $s_p \in \{0,1\}$ displays the membership of the corresponding elementary box in subset B_g^1 or B_g^0 , $R = q_1 \cdot q_2 \cdot ... \cdot q_n$ is the amount of elementary boxes and, consequently, the amount of membership indicators. The one-to-one correspondence between indices $(k_1, k_2, ..., k_n)$ and the index p of an indicator in the set S is defined in (12). It is evident, that zero-based indices are preferable.

$$p = k_1 + q_1 \cdot (k_2 - 1) + q_1 \cdot q_2 \cdot (k_3 - 1) \times ... \\ \times q_1 \cdot q_2 \cdot ... \cdot q_{n-1} \cdot (k_n - 1) \quad (12)$$

Usually, computation of indices $(k_1,k_2,\ldots,k_n)$ for a specified indicator index p is used in the algorithm. The indices are calculated sequentially as it is shown in (13).

$$k_n = \left\lfloor \frac{p-1}{q_1 \cdot q_2 \cdot ... \cdot q_{n-1}} \right\rfloor + 1,$$
$$k_{n-1} = \left\lfloor \frac{p - (k_n - 1) \cdot q_1 \cdot q_2 \cdot ... \cdot q_{n-1}}{q_1 \cdot q_2 \cdot ... \cdot q_{n-2}} \right\rfloor + 1, \quad (13)$$
$$...$$
$$k_1 = p - \sum_{i=2}^{n} \left((k_i - 1) \cdot q_1 \cdot q_2 \cdot ... \cdot q_{i-1}\right), n > 1.$$

The process of the region of acceptability construction on the basis of the model (11) was described in the work [4]. Briefly, this process consists in complete enumeration of the values of index p with calculation of corresponding indices $(k_1, k_2, ..., k_n)$ using (13), calculation its sampling point (8), calculation of membership function (9) and assigning its result to the indicator s_p. The illustration of the result of this process and indicators assignment is presented in Fig.2.

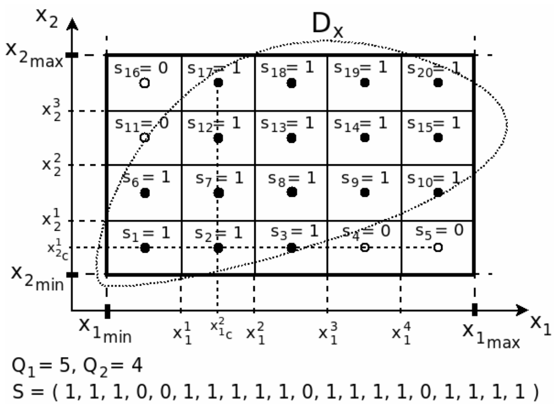

Figure 2. A region of acceptability approximation with a regular grid

The usage of 1-dimensional structure for storing indicators is explained both by the convenience of the data storage and transmission and by its flexibility for task decomposition for parallel processing. The flexibility consists in the opportunity to splitting of the indicators array into arbitrary amount of portions of various volumes (e.g. for load balancing).

III. Parallel Algorithm of Region of Acceptability Construction

One of the advantages of indicator array splitting is the possibility of its parallel processing. As was said before, high computational requirement is a problem of the region of acceptability approximation. Thus, the solving of this task is almost incogitable without parallel computations. The algorithm [4] of region of acceptability approximation on the basis of the model (11) represents the same instructions performed for every elementary box (SPMD – Single Process, Multiple Data). This fact allows for task decomposition on the basis of indicators array splitting.

The construction of the region of acceptability approximation on the basis of the model (11) in parallel processes requires passing the model parameters n, B, Q and the range (14) to the each of P parallel processes.

$$R_i = (p_i^1, p_i^2), i = 1,2,...,P \quad (14)$$

Using these parameters, every process is able to restore univalent indices $(k_1,k_2,\ldots,k_n)$ on the every iteration of indicators enumeration inside the range (14), and, consequently, to calculate the performances (1) and membership function (9). This process is illustrated in Fig. 3.

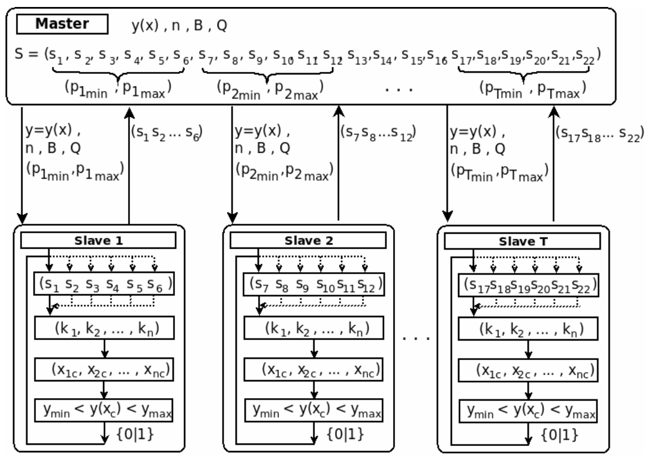

Figure 3. The indicators array decomposition and its parallel processing.

After calculations within the range (14) are performed by the i-th parallel process the part of membership indicators array within the range (14) is filled. The final step of a parallel process work is sending filled part of indicators array to master process which gathers all received parts into complete indicators set $S = (s_1, s_2, ..., s_R)$. Master processor usually can perform computational task as well as slave processes.

IV. Application of Regions of Acceptability to Reliability-Oriented Design

RA can be used for parametric optimization at design stage for researching of parametric deviations. When parameters variation laws are obtained, RA can be used for optimal parameter sizing according to parameter deviation trends in order to provide parametric reliability.

When parameter variation laws are unknown, the most rational way to provide parametric reliability and avoid violation of performance margins (4) is to locate nominal parameters at the most distant point from the border inside of RA as illustrated in Fig. 5.

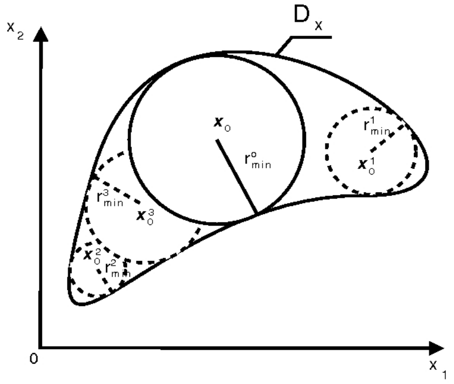

Figure 4. Reliability reserve criterion

According to worst-case principle, a hyper-sphere inscribed into RA provides minimal reserve of parameter variations. Thus, the maximum sphere inscribed into RA provides maximum of all possible minimal reserves of parameter variations.

According to RA approximation method with a discrete set of elementary boxes, boxes enumeration technique should be applied. A figure consisting of elementary boxes should be inscribed into RA approximation i.e. consist of elements from B_g^1 set. The most appropriate figure to inscribe is an r-cube that consists of elementary boxes $e_{k_1k_2...k_n} \in B_g$ with its central element indexed $(k_1^c, k_2^c, ..., k_n^c)$ and other elements with the indices that satisfy (15).

$$k_i^c - r \le k_i \le k_i^c + r, \forall i = 1,2,...,n \quad (15)$$

The example of 2-dimensional r-cube with r=1 is illustrated in Fig.5.

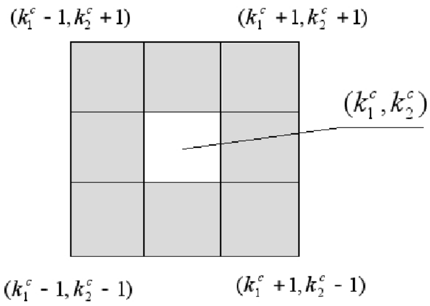

Figure 5. The example of r-cube for r=1

Thus, r-cube is a subset $B_c \subseteq B_g$ of elementary boxes that can be defined with (16).

$$B_c = \{e_{k_1k_2...k_n} \in B_g : k_i^c - r \le k_i \le k_i^c + r\} \quad (16)$$

An additional condition should be added to (16) as evident fact that central element indices to be constrained as shown in (17).

$$2 \le k_i^c \le q_i - 1, \forall i = 1,2,...,n \quad (17)$$

The main idea of inscribing an r-cube of maximal r inside RA is to find such an element $e_{k_1k_2...k_n} \in B_g^1$ inside RA approximation that acts as a central element of maximal r among all elements $e_{k_1k_2...k_n} \in B_g^1$.

The algorithm consists in attempts to construct an r-cube iteratively changing r=1,2,3,… for every subset element $e_{k_1^ck_2^c...k_n^c} \in B_g^1$ that satisfy (17) until at least one of cube elements belongs to B_g^0 subset or $B_c \cap B_g^0 \ne \varnothing$. When a cube with r=1 can not be inscribed its central element $e_{k_1^ck_2^c...k_n^c}$ is assigned an additional weight 0. When a cube with r=1 is successfully inscribed inside B_g^1, an attempt to inscribe a cube with r=2 is performed. If this attempt fails, the central element is assigned an additional weight of 1. If attempt to inscribe a cube with r=2 was successful and r=3 failed, the weight is 2, and so on. On increasing value of r, only border elements of a cube are checked because internal ones have already been checked on previous iterations. As the result of the algorithm every element of B_g^1 is assigned a weight $w_{k_1k_2...k_n} = 0,1,2,...$ of maximal r. The next step is to select an element $e_{k_1k_2...k_n} \in B_g^1$ with maximal weight. In general, the task can be described as maximin task of searching optimal elements with objective function $V(e_{k_1k_2...k_n}, r)$ of r-cube volume what can be expressed in (18).

$$B_{opt} = \{e_{k_1k_2...k_n} \in B_g^1 : \max_{B_g^1} \min_r V(e_{k_1k_2...k_n}, r)\} \quad (18)$$

Usually, there can be more than one element selected. In this case it is required to extract connected subsets of those elements and calculate interpolated coordinates, e.g. arithmetic mean.

Unfortunately, the algorithm of maximal r-cube inscribing has low degree of parallelism and almost can not be implemented in parallel processes but this method uses RA approximation instead of high-cost multiple system simulations (3) in parametric optimization.

V. Software Tools for Regions of Acceptability Construction and Utilization

A software tools implementing algorithms of RA construction and searching for optimal parameters consists of the following main blocks:

- A module for creation task of RA construction.
- Simulation module.
- RA analysis module.

A. A Module for Creation of the Task.

This module allows specifying parameters of RA model (11) and other options of process. System models (3) used by the software are linked to the software as shared libraries. Main parameters of the model are also given in those libraries and imported as defaults. These parameters are:

- System parameters count.
- System parameters constraints (6).
- System performances count.
- System performances constraints (4).

Both parameters constraints and performances constraints can be modified in the main form. Quanta amounts are set by default and should be modified by user. Other options concern indicators set storage. User can choose data compression type and data segmentation that allows performing task in parallel processes and storing data in distributed mode. A screenshot of a user interface of the module is illustrated in Fig. 6.

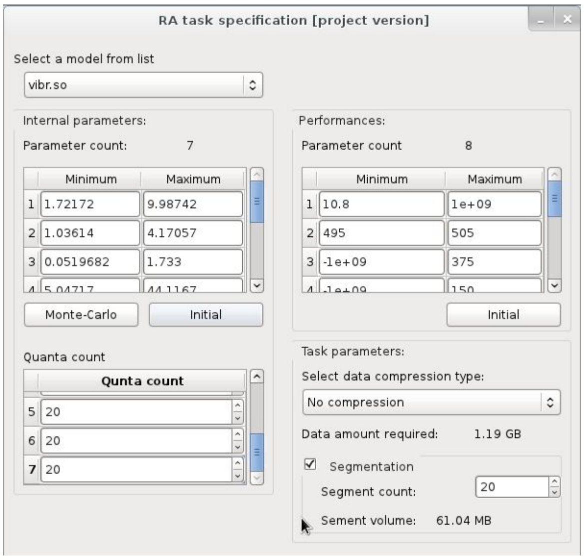

Figure 6. Main windows of a module for RA creation module

B. Simulation Module.

This module performs the algorithm of RA construction. The module fills indicators array within specified range. This module can be performed as one of parallel processes on supercomputer or cluster. The software module performs system simulations using their models implemented in shared and dynamically linked libraries.

C. A Module for RA Analysis

The module allows to solve various task associated with RA analysis. These tasks require constructed RA with description corresponding to model (11). One of the most useful analysis tasks is parameter sizing. This module implements the algorithm of optimal parameter search which has been considered in chapter IV. User may prefer visual analysis of RA. The module allows to visualize RA sections that may include parameter points. The example of 2-dimensional sections is illustrated in Fig. 7.

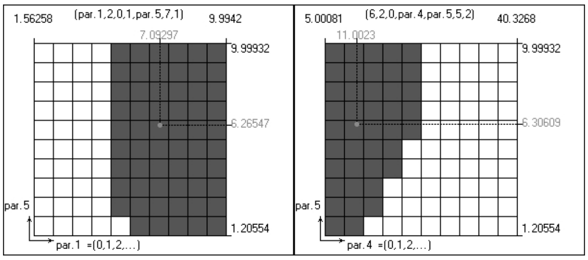

Figure 7. 2-dimensional RA sections with interpolated optimal parameters

VI. CONCLUSIONS

The problem of large amount of data in the framework of the region of acceptability construction on the basis of approximation with a discrete set of elementary boxes is considered in this work. The ways of reducing of the data redundancy and corresponding problem of data access are considered. The methods of increasing random access speed on compressed data are offered. The efficiency of indicators array partitioning both for access speed and for the task decomposition for using of parallel computations technique is presented in this work.

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